Lecture 6. ***ACS with variable structure (SVS). ACS operation modes with variable structure***

6.1. ACS with variable structure (SVS)

To systems with variable structure there refer systems, the structural scheme of which is changing at transmission of representing points through the border of some beforehand established regions on the phase plane. Examples of *the systems with variable structure (SVS)* are relay systems closing or opening a part of the scheme at transmission through the switching line.

The basic idea of making SVS method consists in the following: “stitching together” phase portraits of different structures (using the best properties of each system) we get a new system of a higher level of quality than in any separately taken structure i.e. a system with variable structure. It means that ACS in different time moments works in correspondence with different structures from which it is “stitched”.

Example 6.2. Let us consider two stable linear systems, one of which is characterized by monotonicity of transient process, the other - by high-speed performance. In fig.6.1 and 6.2 there are presented transient processes and phase portraits of the chosen systems.

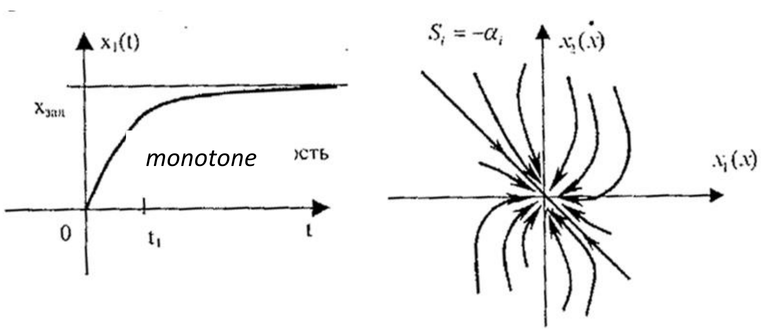

Fig.6.1. Transient process and phase portrait of the first system

From the phase portrait of SVS (Fig.6.3) it is clear, that whereas the initial represented point was situated (system condition) x_0, it transmits to the balance condition point initially by focal point, then by knot.

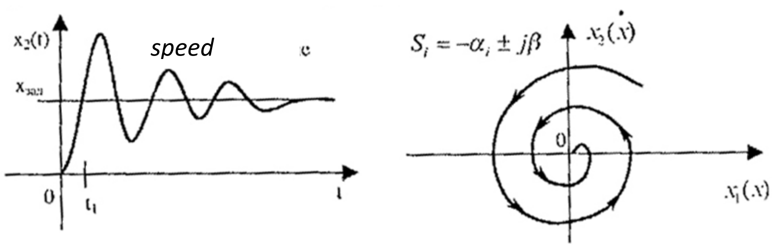

Fig.6.2. Transient and phase portrait of the second system

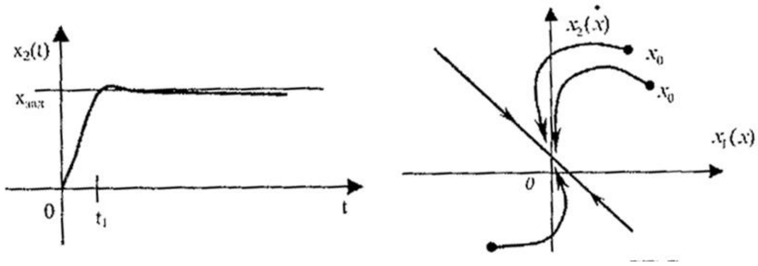

Fig.6.3. Transient and phase portrait of SVS

Example 6.3. Let us consider two instable linear systems, both of them have right poles. Phase portraits of the chosen structures are represented in Fig.6.4: a) saddle type (always instable), b) instable focal point.

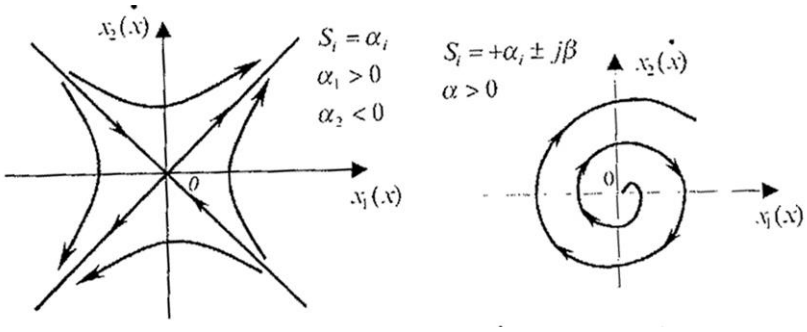

Fig.6.4. Phase portraits of fixed instable structures

By stitching the phase portraits it is possible to get a stable system with nonlinear structure, the phase portrait of which is presented in Fig.6.5.

If motion begins at M_0 point, then it goes firstly by instable phase trajectory of the saddle type, then after crossing $x_2(\dot{x})$ coordinate axis transmits on the trajectory of instable focal point and then by getting on the stable degenerated trajectory, follows it up to the balance point.

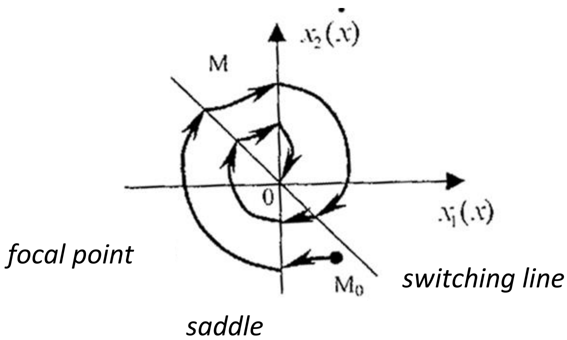

Fig.6.5. A phase portrait of SVS

In *A.A.Voronov's (A.A. Воронов)* works there are given examples of real systems with variable structure, in which transmission from one structure to another is caused by interior physical laws operating in the systems. By introduction into the system switching logical elements, we may artificially get SVS with desired characteristics.

6.2. Working conditions for ACS with variable structure

Let us consider a linearized system of the *2nd* order:

$$\dot{x} = Ax + Bu.$$

The following types of working conditions for systems with variable structure are possible: 1) motion by degenerated phase trajectories condition; 2) switching condition; 3) sliding condition.

SVS consists of two structures. One of the structures must have degenerated trajectories of stable type (mostly degenerated trajectory are straight lines). The other structure must have phase portrait, trajectories of which cross the stable degenerated trajectory.

Let the following structures of the systems be under consideration (fig.6.6).

A system with variable structure will have phase portrait, presented in fig.6.7.

First structure (saddle type) *Second structure (instable focal point)*

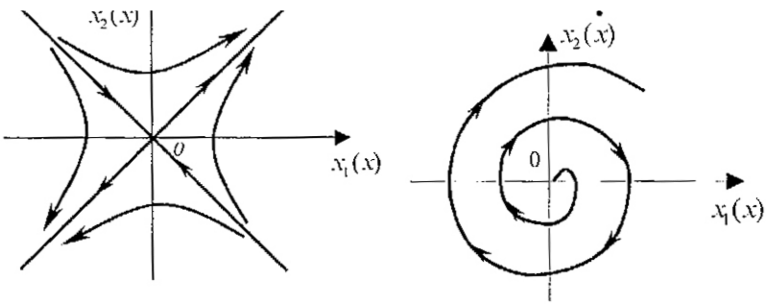

Fig.6.6. A phase portrait of an instable structure

By stitching the phase portraits of instable structures, it is possible to get asymptotically stable SVS phase portrait which is presented in fig.6.24.

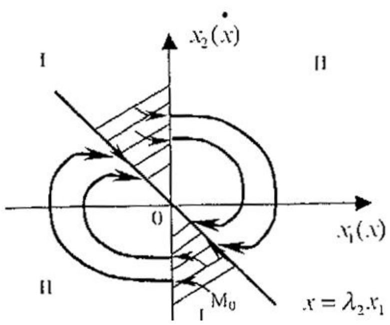

Fig.6.7. A phase portrait of an asymptotically stable SVS

1. The case of motion along the degenerated trajectory condition

Let dynamic system be described by a differential equation of the following type:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -a_1 x_1 - a_2 x_2 + u \end{cases}.$$

Let the control law is the following:

$$u = (m^T x_1) sign\ x_2 ,$$

where *"m"* is a vector of the following type: $m = \begin{vmatrix} m_1 \\ 0 \end{vmatrix}$, and $m_1 = \begin{cases} m_1^1 \text{ - I struuctures;} \\ m_1^2 \text{ - II structures.} \end{cases}$

Then SVS can be presented as in fig. 6.8; here block BChS is a block of changing structure.

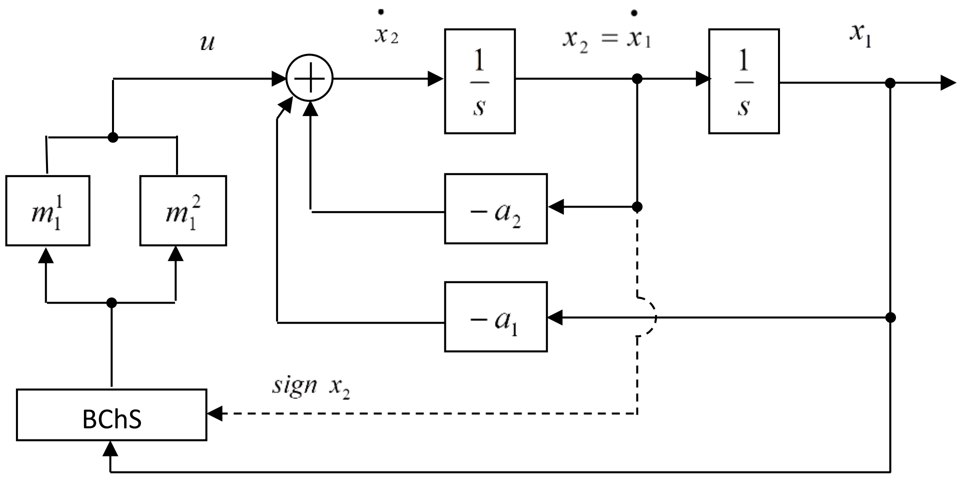

Fig.6.8. Structure of SVS

2. Switching condition

For this condition the special point is the beginning of the coordinates, which makes possible to reach a lot of switching.

Let SVS consists ("is stitched") of two system structures, phase portraits of which have a saddle type (instable) and a type of instable focal point (fig.6.6).

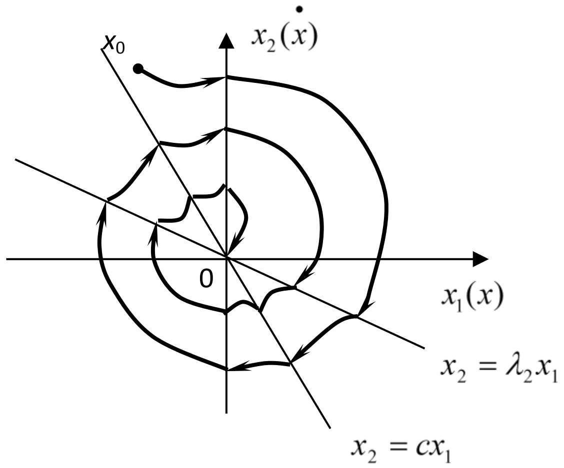

Fig. 6.9. A phase portrait of switching condition

By "stitching" phase portraits, we get SVS, the phase portrait of which is presented in fig. 6.9.

Here the equation of degenerated trajectories has the form:

$$\begin{cases} x_2 = \lambda_2 x_1 \\ x_2 = cx_1, \end{cases}$$

where $|c| > |\lambda_2|$; λ_2 are negative roots of the characteristic equation, nearest to the imaginary axis, $\lambda_2 < 0$; here *"c"* is *const*.

Changes in the motion equation occurs at transmission through degenerated trajectories, which in this case are called *switching lines*, and also through coordinate axis, which in this case is a switching line too.

3. Sliding condition

Let SVS consist ("be stitched") of two structures of systems, the phase portraits of which have a saddle type (instable) and an instable focal point (fig.6.6). The phase portrait of SVS is presented in fig.6.10.

The equations of degenerated phase trajectories have the following form:

$$\begin{cases} x_2 = cx_1 \\ x_2 = \lambda_2 x_1 \end{cases}, \text{ but } |c| < |\lambda_2|.$$

To the switching line $x_2 = cx_1$ the phase trajectory approaches from two sides. After getting on the switching line, representing point cannot leave it, but also cannot stay on it (it is pushed out from the both sides, because the trajectories are *directed to meet each other* (fig.6.10.) on segment *AB).*

The speed of the motion on *AB* is not determined, but special research shows, that it is finite and its size oscillates near the value of the point ordinate. Representing point will slide along the segment to the beginning of coordinates, i.e. to the balanced point. Segment *AB* is called *a sliding line*. SVS condition in the case when representing point is moving along the switching line, where changing of the structure occurs with infinitely big frequency, is called *sliding*.

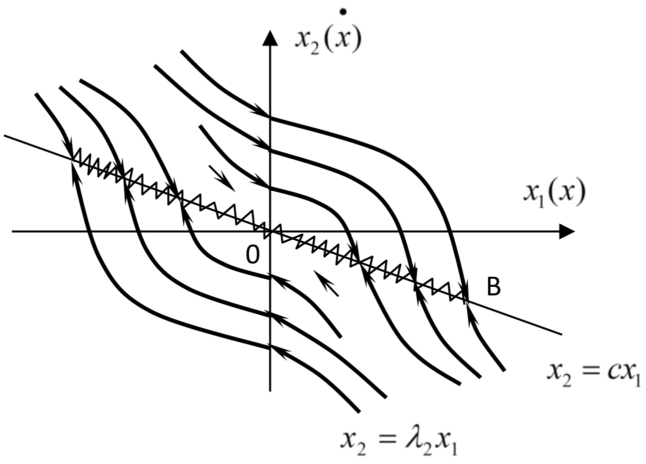

Fig. 6.10. The SVS phase portrait

So, the main idea of a method of creating SVS consists in the following: "sewing" phase portraits of different structures (*at the same time the best properties of each of systems undertake*); we create more quality new system, than in any of single structures – a system with variable structure. It means that ACS in different time points works according to different structures from which it "is sewed". What we were quite convinced of.